Superconformal Gauge Theories and Non-Critical Superstrings

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Abstract

We consider effective actions for six-dimensional non-critical superstrings. We show that the addition of N units of R-R flux and of N_f space-time filling D5-branes produces $AdS_5 \times S^1$ solutions with curvature comparable to the string scale. These solutions have the right structure to be dual to $\mathcal{N} = 1$ supersymmetric $SU(N)$ gauge theories with N_f flavors. We further suggest bounds on the mass-squared of tachyonic fields in this background that should restrict the theory to the conformal window.

arXiv:hep-th/0409133v4 9 Dec 2004

1 Introduction

Most known examples of the AdS_5/CFT_4 correspondence [1, 2, 3] relate conformal 4-d gauge theories to $AdS_5 \times X^5$ backgrounds of the *critical* type IIB strings with R-R flux. This correspondence may be motivated by considering stacks of D3-branes and taking the low-energy limit [4]. When the 't Hooft coupling of the gauge theory is made large, the curvature becomes small in the dual description, so that many calculations can be performed using the supergravity approximation to the type IIB string theory. This duality provides a great deal of information about strongly coupled gauge theory. However, the simplest methods apply only to theories that can be taken to very strong 't Hooft coupling. One expects that there are many superconformal gauge theories that do not satisfy this rather stringent requirement. In fact, many of the $\mathcal{N} = 1$ superconformal gauge theories discovered by Seiberg [5] do not have known AdS duals.

For this reason it is interesting to search for string theories other than the conventional critical ones, as starting points for the AdS/CFT duality. In this connection Polyakov proposed to use non-critical type 0 string theory [6]. By considering the effective action of this theory, he provided evidence for the existence of an AdS_5 solution whose dual should be a non-supersymmetric conformal gauge theory. Furthermore, there exist non-critical string solutions of the form $AdS_p \times S^k$, $k > 1$, supported by R-R flux [6, 7], and AdS_p times tori [8].[1] The curvature of the AdS space, however, turns out to be of order the string scale, so there may be large corrections to the 2-derivative effective action approximation. Nevertheless, the effective action may still lead to interesting qualitative information about the dual gauge theory.

In this note we consider a generalization of the effective action approach to $\mathcal{N} = 1$ superconformal gauge theories. Since such theories have a $U(1)$ R-symmetry, the simplest geometry they may be dual to is $AdS_5 \times S^1$, with the R-symmetry realized by translations in S^1. A sigma model construction of this background, as well as of AdS backgrounds in other dimensions, was recently presented in [10]. In search of $AdS_5 \times S^1$ geometries, we consider backgrounds created by D-branes in the 6-dimensional non-critical superstring theory. Non-critical superstrings were studied in [11, 12, 13, 14] and recently reviewed in [15]. The 6-dimensional case is of particular interest. In the absence of D-branes, it is believed [12, 13] that its background $R^{3,1} \times SL(2,R)/U(1)$ is dual to type IIB string theory on the resolved conifold; the mirror background $R^{3,1}$ times $\mathcal{N} = 2$ super-Liouville theory is dual to IIB string theory on the deformed conifold.

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[1]Also, backgrounds of the form AdS_p times linear dilaton were recently found in [9].

Starting with the type IIB theory, the $\mathcal{N} = 1$ supersymmetric $SU(N)$ gauge theory may be realized on N D5-branes wrapped over the 2-cycle of the resolved conifold. The backgrounds they create were studied in [16, 17, 18] where it was was demonstrated that the theory makes a geometric transition to the deformed conifold. Furthermore, by adding D5 [19, 20, 21] or D7 branes [22, 23, 24, 25, 28, 29], it is possible to introduce flavors into the gauge theory. In this paper we use similar methods to embed $\mathcal{N} = 1$ SYM theory into the 6-dimensional non-critical superstring. The theory without matter may be realized on a stack of N D3-branes of the 6-d superstring placed at the tip of the $SL(2, R)/U(1)$ cigar. The flux produced by these D3-branes warps and deforms the $R^{3,1} \times SL(2, R)/U(1)$ geometry; we study its effect using the minimal 2-derivative effective action of the 6-d non-critical superstring. We do not find an $AdS_5 \times S^1$ solution; this could be expected since there is no candidate superconformal gauge theory without flavors. This motivates us to add N_f space-time filling uncharged D5 branes which realize N_f flavors in the SYM theory. This system possesses $AdS_5 \times S^1$ solutions for all values of $\frac{N_f}{N} > 0$. We further suggest that requiring the mass-squared of open string tachyons to lie in the range $-4 < (mR_{AdS})^2 < -3$, where both the Δ_+ and the Δ_- quantizations are admissible [30, 31], restricts these backgrounds to the Seiberg conformal window [5].

A circumstance that complicates our analysis is that, as in [6], the curvature of AdS_5 is of order the string scale (with the 2-derivative effective action, we find that $R_{AdS}^2 = 6\alpha'$ for all N and N_f). Therefore, our results are expected to receive $O(1)$ corrections from higher-derivative terms. Nevertheless, it is plausible that such corrections do not destroy our $AdS_5 \times S^1$ solutions.

2 Effective Action of the Non-Critical String

Let us consider the following effective action for the 6-d non-critical superstring theory,

$$S \sim \int d^6 x \sqrt{-G} [-(\partial_\mu \chi)^2 + e^{-2\phi} (R + 4(\partial_\mu \phi)^2 + \lambda^2) - 2N_f e^{-\phi}] , \qquad (2.1)$$

where χ is a R-R scalar dual to the 5-form field strength.[2] The cosmological constant is

$$\lambda^2 = \frac{10 - d}{\alpha'} = 4 ,$$

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[2] We did not compute the precise numerical coefficients in the normalization of the RR terms and of the D5-brane tension.

in units where $\alpha' = 1$. We will assume that $\chi = N\theta$. This means that we have $O(N)$ units of RR flux, so the number of colors in the gauge theory is proportional to N. The last term in (2.1) is due to uncharged D5-branes, which can be thought of as D5/anti-D5 pairs, filling all six space-time dimensions. So, the parameter N_f is proportional to the number of massless flavors in the gauge theory.

Let us transform to the Einstein metric,

$$g_{\mu\nu} = e^{-\phi} G_{\mu\nu} \ . \tag{2.2}$$

The action becomes

$$\int d^6x \sqrt{-g} [-e^{2\phi}(\partial\chi)^2 + R - (\partial\phi)^2 + e^\phi \lambda^2 - 2N_f e^{2\phi}] \ . \tag{2.3}$$

We adopt the following ansatz for the 6-d Einstein metric:

$$ds_E^2 = \left[e^{-2B(u)/3}(du^2 + e^{2A(u)}dx_i dx_i) + e^{2B(u)}d\theta^2 \right] , \tag{2.4}$$

Substituting this into the action, we find

$$S \sim \int du\, e^{4A}[3(A')^2 - \frac{1}{3}(B')^2 - \frac{1}{4}(\phi')^2 + \frac{1}{4}\lambda^2 e^{\phi-(2B/3)} - \frac{1}{4}N^2 e^{2\phi-(8B/3)} - \frac{N_f}{2}e^{2\phi-(2B/3)}] \ . \tag{2.5}$$

When the action takes the form

$$S = \int du\, e^{4A}\left[3A'^2 - \frac{1}{2}\mathbf{G}_{ab}(f)f'^a f'^b - V(f) \right] , \tag{2.6}$$

there is the following prescription for looking for first-order equations [32, 33, 34]. If we find the superpotential W, then

$$f'^a = \frac{1}{2}\mathbf{G}^{ab}\frac{\partial W}{\partial f^b} , \qquad A' = -\frac{1}{3}W(f) , \tag{2.7}$$

where the superpotential W is a function of scalars f^a satisfying

$$V = \frac{1}{8}\mathbf{G}^{ab}\frac{\partial W}{\partial f^a}\frac{\partial W}{\partial f^b} - \frac{1}{3}W^2 \ . \tag{2.8}$$

Here we have only two scalars: B and ϕ.

$$\mathbf{G}_{\phi\phi} = \frac{1}{2} , \mathbf{G}_{BB} = \frac{2}{3} \ . \tag{2.9}$$

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The potential is

$$V = -\frac{\lambda^2}{4}e^{\phi-(2B/3)} + \frac{N^2}{4}e^{2\phi-(8B/3)} + \frac{N_f}{2}e^{2\phi-(2B/3)} \ . \tag{2.10}$$

If $N = N_f = 0$ then the superpotential may be taken of the form

$$W \sim \lambda \exp[(\phi/2) - (B/3)] \ . \tag{2.11}$$

This gives a system of first-order equations. The resulting solution is that the string frame metric is exactly flat and the dilaton linear

$$ds^2 = dx_i dx_i + d\rho^2 + d\theta^2 \ , \tag{2.12}$$
$$\phi = -\rho \ , \tag{2.13}$$

where $\rho = \frac{3}{\lambda}\log u$.

To get the well-known "cigar" solution [35] times R^4, we need to adopt a different superpotential:

$$W = -\exp[-4B/3] - \exp[\phi + (2B/3)] \ , \tag{2.14}$$

which also gives $V = -\exp[\phi - (2B/3)]$. We define a new variable g and a new radial variable r through

$$dr = \exp[(\phi/2) - (B/3)]du \ , \tag{2.15}$$
$$g = \frac{\phi}{2} + B \ .$$

We see that e^g is the radius of the circle in string units. We find the following set of first order equations:

$$\frac{\partial\phi}{\partial r} = -e^g \ , \tag{2.16}$$
$$\frac{\partial g}{\partial r} = -e^g + e^{-g} \ . \tag{2.17}$$

They have a solution

$$e^{-\phi} = \cosh r \ , \qquad e^g = \tanh r \ . \tag{2.18}$$

So the string frame metric is

$$ds^2 = dx_i dx_i + dr^2 + \tanh^2 r d\theta^2 \ , \tag{2.19}$$

which is precisely the 2-d black hole metric.

3 The $AdS_5 \times S^1$ Solutions

For general N and N_f there exists an $AdS_5 \times S^1$ solution where the dilaton and the radius of S^1 have u-independent values which extremize the effective potential:

$$\frac{\partial V}{\partial \phi} = \frac{\partial V}{\partial B} = 0 \ . \tag{3.20}$$

These equations are

$$\frac{2}{3}e^{\phi-(2B/3)} - \frac{2N^2}{3}e^{2\phi-(8B/3)} - \frac{N_f}{3}e^{2\phi-(2B/3)} = 0 \ , \tag{3.21}$$

$$- e^{\phi-(2B/3)} + \frac{N^2}{2}e^{2\phi-(8B/3)} + N_f e^{2\phi-(2B/3)} = 0 \ . \tag{3.22}$$

The solution is

$$e^{2B} = \frac{N^2}{N_f} \ , \qquad e^\phi = \frac{2}{3N_f} \ . \tag{3.23}$$

Now, we define the variables r and g as above, (2.15), and

$$f = \frac{\phi}{2} - \frac{B}{3} + A \ . \tag{3.24}$$

In terms of these new variables the ansatz for the string metric is

$$ds^2 = e^{2f}dx_i dx_i + dr^2 + e^{2g}d\theta^2 \ . \tag{3.25}$$

From the constraint associated with the action (2.6),

$$3\left(\frac{\partial A}{\partial u}\right)^2 - \frac{1}{2}\mathbf{G}_{ab}(f)\frac{\partial f^a}{\partial u}\frac{\partial f^b}{\partial u} + V(f) = 0 \ , \tag{3.26}$$

we find

$$\left(\frac{\partial f}{\partial r}\right)^2 = \frac{1}{6} \ . \tag{3.27}$$

This implies that in the string frame the AdS radius is $\sqrt{6\alpha'}$. As $N_f \to 0$ this solution becomes singular. Indeed, for $N_F = 0$ and $N > 0$ there is no $AdS_5 \times S^1$ solution of the equations, since (3.21) and (3.22) are incompatible.

The solution above could also be found directly from the Einstein equations. Starting with the Einstein frame, we find that the dilaton variational equation is

$$2e^{2\phi}[2N_f + (\partial\chi)^2] = 4e^\phi \ , \tag{3.28}$$

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while the Einstein equation is

$$R_{\mu\nu} = \frac{1}{2}g_{\mu\nu}(R - e^{2\phi}(\partial\chi)^2 - 2N_f e^{2\phi} + 4e^{\phi}) + e^{2\phi}\partial_{\mu}\chi\partial_{\nu}\chi \ . \tag{3.29}$$

It follows that

$$R = e^{2\phi}(\partial\chi)^2 + 3(N_f e^{2\phi} - 2e^{\phi}) \tag{3.30}$$

and

$$R_{\mu\nu} = \frac{1}{2}g_{\mu\nu}(N_f e^{2\phi} - 2e^{\phi}) + e^{2\phi}\partial_{\mu}\chi\partial_{\nu}\chi \ . \tag{3.31}$$

Now it is easy to check that the RHS of the equation for $R_{\theta\theta}$ is indeed zero when we substitute $\chi = N\theta$ and the solution (3.23) for B and ϕ. The dilaton equation is also satisfied.

The equation for the remaining components R_{ij} indicate that it is a space of constant negative curvature:

$$R_{ij} = -\frac{4}{9N_f}g_{ij} \ . \tag{3.32}$$

After we transform back to the string frame, we have

$$R_{ij} = -\frac{4}{6}G_{ij} = -\frac{4}{R_{AdS}^2}G_{ij} \ . \tag{3.33}$$

Thus, all dependence of the AdS_5 radius on N_f and N cancels out, and we again find that it is $\sqrt{6\alpha'}$.[3] In summary, for this solution the string frame radii of the S^1 and the AdS_5 are

$$\frac{R_{S^1}^2}{\alpha'} = e^{2g} = \frac{2}{3}\frac{N^2}{N_f^2} \ . \tag{3.34}$$

$$\frac{R_{AdS}^2}{\alpha'} = 6 \ . \tag{3.35}$$

The 5-dimensional effective coupling is determined by

$$\frac{1}{g_5^2} = e^{-2\phi}R_{S^1} = \frac{3\sqrt{3\alpha'}}{2\sqrt{2}}NN_f \ . \tag{3.36}$$

It is of further interest to calculate the masses of the fluctuations in ϕ and B around the minimum (3.23) of the potential. We find that N_f and N scale out of the calculation, so that the masses are completely independent of N_f and N:

$$m_1^2 = \frac{5 + \sqrt{13}}{3\alpha'} \ , \qquad m_2^2 = \frac{5 - \sqrt{13}}{3\alpha'} \ . \tag{3.37}$$

[3]Note that this result does not depend on the numerical coefficients in the RR flux quantizations, which we have not fixed.

The corresponding fields are certain mixtures of ϕ and B. Since both m^2 are positive, the corresponding operator dimensions are determined by the positive branch, $\Delta_+ = 2 + \sqrt{4 + (mR_{AdS})^2}$, and we find

$$\Delta_1 = 3 + \sqrt{13} \,, \qquad \Delta_2 = 1 + \sqrt{13} \,. \tag{3.38}$$

The dimensions are not rational, which is likely due to the fact that our solution undergoes $O(1)$ corrections by higher-derivative terms.

4 First Order Equations

There exists a simple first order formulation of the equations we need to solve. We consider the following superpotential:[4]

$$W = -\frac{N}{N_f} \exp[-4B/3] - \frac{N_f}{N} \exp[\phi + (2B/3)] + N \exp[\phi - (4B/3)] \,. \tag{4.39}$$

Using (2.8) and (2.9) we get the potential (2.10).

From the superpotential (4.39) we get the first-order equations

$$\frac{\partial g}{\partial r} = \frac{N}{N_f} e^{-g} - \frac{N_f}{N} e^g - \frac{1}{2} N e^{\phi - g} \,, \qquad \frac{\partial \phi}{\partial r} = -\frac{N_f}{N} e^g + N e^{\phi - g} \,, \qquad \frac{\partial f}{\partial r} = \frac{1}{2} N e^{\phi - g} \,. \tag{4.40}$$

We note that there exists an $AdS_5 \times S^1$ solution with $e^{2g_c} = \frac{2N^2}{3N_f^2}$ and $N_f e^{\phi_c} = \frac{2}{3}$. This means that $e^{2B_c} = \frac{N^2}{N_f}$.

It is possible to study the flows associated to the equation (4.40). We were not able to find them analytically. As a first step we can perturb the equation around the point at (g_c, ϕ_c). Deviations around this AdS fixed point are governed by a linear equation for $(\delta g, \delta \phi)$ given in terms of a matrix

$$M = \sqrt{\frac{2}{3}} \begin{pmatrix} -2 & -\frac{1}{2} \\ -2 & 1 \end{pmatrix} \tag{4.41}$$

with eigenvalues

$$\lambda_\pm = \frac{1}{\sqrt{6}} \left(-1 \pm \sqrt{13} \right) \tag{4.42}$$

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[4]In the original version of this paper the superpotential was written down for $\frac{N_f}{N} = 1$. We thank Stanislav Kuperstein and Cobi Sonnenschein for sending us its generalization to arbitrary $\frac{N_f}{N}$.

We see that λ_+ is positive but λ_- is negative. Defining the standard radial variable $z = e^{-r/\sqrt{6}}$ such that the AdS metric is $6(dz^2 + dx^2)/z^2$, we find that one of the perturbations grows in the UV as $z^{-\sqrt{6}\lambda_+}$. Comparing with the operator dimensions (3.38) we note that this behavior is $z^{4-\Delta_1}$ corresponding to adding an irrelevant operator to the action. The other perturbation instead grows in the IR as $z^{-\sqrt{6}\lambda_-}$. This behavior is z^{Δ_2} corresponding to giving an expectation value to an operator of dimension Δ_2 [31].

If we integrate the flow starting from the AdS region at very negative values of r, then we need to move away from the IR fixed point in the direction corresponding to the eigenvector of M with positive eigenvalue, i.e. we forbid the perturbation that grows in the IR as z^{Δ_2}. Integrating the equations numerically we find that this leads to the cigar solution at large values of r. We see a plot of the resulting function in figure 1. In conclusion, there is a unique flow between the linear dilaton region and the AdS region.
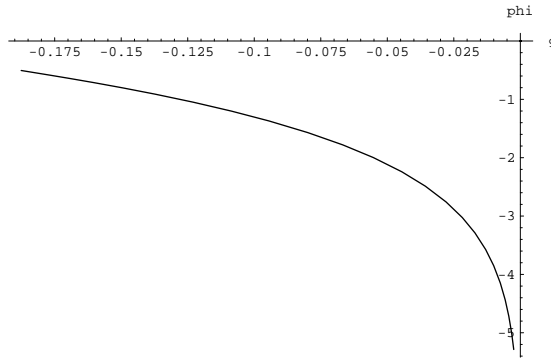

Figure 1: Flow between the $AdS_5 \times S^1$ and $R^4 \times$(linear dilaton). The horizontal axis is g and the vertical axis is ϕ. In the upper left-hand side the flow reaches the AdS fixed point; in the low right-hand side it asymptotes to the vertical line. Thus, in the UV $g \to 0$ and the coupling becomes very weak.

5   Stability, Unitarity and the Conformal Window

The stability of our $AdS_5 \times S^1$ solutions is an important issue since the theory contains fields with $m^2 < 0$, and it is important to enforce the Breitenlohner-Freedman [30] condition $m^2 R^2_{AdS} \geq -4$. Indeed, the theory on the cigar with the appropriate GSO projections (see [15] for a discussion) contains the ordinary tachyon with one unit of

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momentum or one unit of winding on the cigar. Our first task is to calculate the effective mass of these modes in our solutions.

Let us first assume the minimal string frame action for the tachyon:

$$-\frac{1}{2} \int d^6 x \sqrt{-G} [e^{-2\phi}((\partial_\mu T)^2 + m_0^2 T^2)] \ , \tag{5.43}$$

where $m_0^2 = -2$ (in units where $\alpha' = 1$). The masses of the tachyons with one unit of momentum and one unit of winding are:

$$m_{1,0}^2 = -2 + \frac{1}{R_{S^1}^2} \ , \qquad m_{0,1}^2 = -2 + R_{S^1}^2 \tag{5.44}$$

Since $R_{AdS}^2 = 6$, we see that for any R_{S^1} the winding and/or the momentum mode violates the B-F bound. This seems to lead to the unfortunate conclusion that the solutions we found are unstable.

We believe that the solution of this problem is related to non-minimal couplings of the tachyon field. Indeed, additional terms coupling the tachyon to R-R fields were found in [36]. In the 6-d string case the extra term must have the form

$$-\frac{1}{2} \int d^6 x \sqrt{-G} b |F_5|^2 T^2 \ , \tag{5.45}$$

where b is a normalization constant. Dualizing F_5 to the R-R scalar, we see that the action becomes

$$-\frac{1}{2} \int d^6 x \sqrt{-G} [e^{-2\phi}((\partial_\mu T)^2 + m_0^2 T^2) + b(\partial_\mu \chi)^2 T^2] \ . \tag{5.46}$$

The last term causes a shift of the effective mass to

$$m_{eff}^2 = m_0^2 + be^{2\phi} G^{\theta\theta} N^2 = m_0^2 + be^{\phi - 2B} N^2 = -2 + \frac{2b}{3} \ . \tag{5.47}$$

The masses of the tachyons with one unit of momentum and one unit of winding are:

$$m_{1,0}^2 = m_{eff}^2 + \frac{1}{R_{S^1}^2} \ , \qquad m_{0,1}^2 = m_{eff}^2 + R_{S^1}^2 \tag{5.48}$$

We see that, if $b \geq 2$, then $m_{eff}^2 \geq -2/3$ and the B-F bound is satisfied for any R_{S^1}. In this case the stability imposes no restriction on N_f/N.

A different situation emerges for $b < 2$. Consider, for example, $b = 1$. Then the tachyons satisfy the Breitenlohner-Freedman bounds as long as

$$\frac{1}{R_{S^1}^2} \geq \frac{2}{3} \ , \qquad R_{S^1}^2 \geq \frac{2}{3} \ , \tag{5.49}$$

where the first comes from the momentum, and the second from the winding mode. Thus, the space is stable in the range

$$\frac{3}{2} \geq R_{S^1}^2 \geq \frac{2}{3} \ , \tag{5.50}$$

which translates into

$$1 \leq \frac{N}{N_f} \leq \frac{3}{2} \ . \tag{5.51}$$

This results in a restriction on the gauge theory that is reminiscent of the conformal window of [5],[5] but we actually do not expect the closed string tachyons to create an instability, i.e. we expect that $b \geq 2$.

The conformal window restriction should come from considering the open string tachyons on the D5-branes because they are expected to be dual to meson operators $Q^i \tilde{Q}_j$, where i, j are flavor indexes. Each meson superfield contains two scalar operators: the squark bilinear of dimension $\Delta_1 = 3\frac{N_f - N_c}{N_f}$, and the quark bilinear of dimension $\Delta_2 = 1 + 3\frac{N_f - N_c}{N_f}$. At the lower edge of the conformal window, $N_f = 3\frac{N_c}{2}$, $\Delta_1 = 1$, saturating the unitarity bound [5]. Within the conformal window, $\frac{3N_c}{2} < N_f < 3N_c$, we find $1 < \Delta_1 < 2$ and $2 < \Delta_2 < 3$.

Now, let us consider the dual $SU(N_f - N_c)$ magnetic theory which contains N_f flavors q^i, $\tilde{q}_j$, as well as gauge singlet free superfields M_i^j, with superpotential $M_i^j q^i \tilde{q}_j$. In the UV theory the superfields $q^i \tilde{q}_j$ contain scalar operators of dimensions $3\frac{N_c}{N_f}$ and $1 + 3\frac{N_c}{N_f}$. To flow to the IR fixed point we have to integrate over the superfields M_i^j. This introduces a Legendre transform which changes the dimension of an operator from Δ to $4 - \Delta$ [31]. Therefore, in the infrared theory we find scalar operators of dimension $1 + 3\frac{N_f - N_c}{N_f}$ and $3\frac{N_f - N_c}{N_f}$, in complete agreement with what was found from the electric point of view.

Now let us see what these well-known field theory results [5] may imply about the dual string theory in AdS space. The AdS/CFT correspondence associates a scalar operator of dimension Δ with a field in AdS_5 of $(mR_{AdS})^2 = \Delta(\Delta - 4)$. Substituting the dimensions Δ_1, Δ_2 we find

$$(m_1 R_{AdS})^2 = \left(3\frac{N_c}{N_f} - 1\right)^2 - 4 \ , \qquad (m_2 R_{AdS})^2 = \left(2 - 3\frac{N_c}{N_f}\right)^2 - 4 \ . \tag{5.52}$$

These formulae have an interesting structure. We observe that the Seiberg duality transformation $N_c \to N_f - N_c$ interchanges $(m_1 R_{AdS})^2$ and $(m_2 R_{AdS})^2$. Since Δ_1 lies

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[5]The Seiberg window [5] is $\frac{1}{3} < \frac{N_c}{N_f} < \frac{2}{3}$.

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in the "negative branch" range [31] (between 1 and 2), while Δ_2 lies in the "positive branch" range (between 2 and 3),[6] the Seiberg duality in effect changes the quantization conditions for each field: $\Delta_i \to 4 - \Delta_i$, $i = 1, 2$.[7] For this operation to be admissible, it is necessary that m_1^2, m_2^2 lie in the range where both the Δ_+ and the Δ_- quantizations ($\Delta_\pm = 2 \pm \sqrt{4 + (mR_{AdS})^2}$) are allowed [30, 31]:

$$- 4 < (mR_{AdS})^2 < -3 . \tag{5.53}$$

Outside of this range Δ_- violates the unitarity bound. Note also that at the edges of the conformal window one of the two masses saturates the BF bound. Imposing the condition (5.53) on (5.52) we indeed find that N_f/N_c is restricted to the conformal window. We propose therefore that, if the complete string dual of the $SU(N_c)$ superconformal gauge theories with N_f flavors is constructed, then the condition (5.53) on the open string tachyons will restrict it to the conformal window.

The specifics of this construction remain to be worked out, of course. In particular, it is not clear why the open string tachyon, whose bare $m^2 = -\frac{1}{2\alpha'}$, i.e. $m^2 R_{AdS}^2 = -3$, can approach the BF bound. Perhaps the m^2 receives a negative shift due to interaction with the R-R fluxes or α' corrections. In any case, it is clear that the 2-derivative effective action is mainly a qualitative tool, and that a more precise treatment of D-branes on the cigar is necessary. Another important difficulty is the fact that there is a large number of flavor branes, which can lead to strong coupling effects for open strings in the bulk. In other words, we find that $e^\phi N_f \sim \frac{2}{3}$, which is not small.

Finally, we would like to speculate that T-duality on the circle S^1 is related to Seiberg's electric-magnetic duality [5]. This is suggested by eq. (5.52) and the discussion below it. It is tempting to think that m_1^2 and m_2^2 refer to states of the open string tachyon with some momentum or some winding. Then it is natural that such states are interchanged under the T-duality.[8] In the 2-derivative effective action approximation, after the T-duality we have a solution of the same form as the one we started with but

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[6]These properties of the meson operator dimensions were pointed out in [37].

[7]For $N_f = 2N_c$, where the Seiberg duality maps the gauge theory into itself, we find $(m_1 R_{AdS})^2 = (m_2 R_{AdS})^2 = -15/4$. One of these fields corresponds to an operator of dimension $\Delta_- = 3/2$, while the other to $\Delta_+ = 5/2$. Two scalar fields with $(mR_{AdS})^2 = -15/4$ are found in the spectrum of IIB supergravity on $AdS_5 \times T^{11}$, which is dual to the $SU(N) \times SU(N)$ gauge theory on D3-branes at the tip of the conifold [31]. This gauge theory is also mapped into itself by the Seiberg duality, and the $SU(N)$ theory with $2N$ flavors is obtained when we take the coupling of one of the gauge groups to zero. In this limit the radius of $AdS_5 \times T^{11}$ becomes small, and the supergravity approximation breaks down.

[8]For this it is necessary that the branes introducing flavors are themselves invariant under the T-duality. Therefore, the correct string construction should probably include D4-branes and D5-branes.

with $N_f \to \sqrt{\frac{2}{3}}N$ and $N \to \sqrt{\frac{3}{2}}N_f$. This transformation preserves both the radius of the AdS_5 and the effective 5-d coupling. However, it is not of the same form as the Seiberg duality [5] because the latter interchanges N_c with $N_f - N_c$. It is amusing to observe that, if we denote $N_c = \sqrt{\frac{2}{3}}N$, and replace N_f by $N_f - \sqrt{\frac{2}{3}}N$ throughout our paper, then the T-duality transformation becomes identical to the Seiberg duality. We hope that a better analysis of D-branes on the cigar could explain the required shift of N_f.

Acknowledgments

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